Limitations in Total System Weight for Solar Aircrafts Designed for Infinite Flight

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***Abstract* – This paper presents a feasibility criterion in weight for solar powered aircrafts designed for infinite flight. The criterion is fast and simple to compute and can be used for initial assessments on aircraft design and mission planning. Different approaches in modelling coming from a solar-cell weight, an energy-storage weight and a propulsion system weight perspective led to different limitations. Depending on what modeling approach was used, it can also be seen whether the aircraft is capable of infinite level flight or if energy has to be stored in altitude.**

The hereby presented criterion was validated using two very different solar aircrafts, SkySailor, a 3.2m wingspan, 2.55kg small type of airplane, and the Helios HP01 Prototype, a 75.3m, 929kg one of the largest solar aircrafts. Both validations proved the concept and furthermore helped to estimate the performance of SkySailor off design point and to estimate some, to us unknown, quantities of Helios.

A. Symbols

$P_{req,min}$	Minimal Power requirement for level flight
c_{D0}	Wing drag coefficient at zero lift
e	Wing Osswald efficiency
Λ	Wing aspect ratio
g	Gravitational acceleration
b	Wingspan
ρ	Air density
k	Increasing factor
m	Mass
P	Power
E	Energy
A	Area
σ	Specific weight
η	Efficiency
I	Intensity

I. INTRODUCTION

In complex engineering disciplines, fast and simple tools to check the feasibility of a system design can save enormous efforts. Recently, there have been large progresses in the field of solar flight, first over night flights have been performed by both, unmanned and manned systems. Whilst one of the manned systems, solar impulse, is currently performing nearly 1000mi distances, unmanned systems have reached continuous flight of up to 14 days. In order to push these limits and reach infinite flight with solar airplanes, solar aircraft systems get particularly complex and require interdisciplinary high engineering performances. In such an environment, this paper presents a tool to fast and simply compute a system weight limit for solar aircrafts performing infinite flight and using solar power as their only energy source.

The approach is to assume a perfectly balanced, working system in horizontal stationary flight, where lift equals weight, drag equals thrust and especially, the energy generated during the daylight equals the total energy consumption of the system in 24 hours. Then, the systems mass is increased by an arbitrary value to see, what additional energy demand would arise due to this new weight and what the power generating weight increase would be, in order to compensate this new demand. This will be an iterative circle, as added power generating weight will again raise the systems energy demand. The iterative circle can be stable, if it converges to a newly balanced system weight or unstable, if the added weight leads to higher and higher power generating weights. It was found out, that this consideration can only be stable for a distinct region, leading to a limit in total system weight for solar airplanes.

B. Related Work

Colozza [1] and Craig [5] presented an initial feasibility study, an analysis of alternatives and technical requirements for High Altitude Long Endurance (HALE) solar airplanes at NASA, based on the Helios HP01 and HP03 prototypes recent altitude record flight (96’863ft, 2001). Hepperle [6] did a similar study at the DLR. Keidel [3] and Noth [4] both developed a design tool for solar HALE systems performing infinite flight, where the systems Solitair respectively SkySailor resulted out of. Leutenegger, Siegwart and Jabas [2] deepened this work presenting a multi-disciplinary optimization tool for solar flight. Rapinett [9] presented Qinetiqs Zephyr, an endurance record (2011) flying solar UAV. And Xian-Zhong et al. [7] very recently showed a way to determine intial parameters for those design concepts.

C. Contribution

Up to this point, design tools were developed and ways to determine initial parameters were shown. This paper takes the opposite approach and shows what the limits of solar HALE technology are in order to do feasibility assessments of given missions in a fast and simple way.

II. DERIVATION

A. Solar Power Considerations

First, we assume a perfectly balanced solar airplane in stationary level flight, where balanced in particular means that the energy generated during the day equals the total energy consumption of the system. The minimal power required [3], [4] for such a system in level flight can be calculated as follows,

$$P_{req,min} = 4 \cdot \sqrt[4]{\frac{4 \cdot c_{D0}}{27\pi^3 e^3 \Lambda} \frac{(mg)^{\frac{3}{2}}}{b\sqrt{\rho}}} \tag{1}$$

We can now define an arbitrary mass increase

$$m_{new} = (1+k)m_{prior} \tag{2}$$

whereas

$$\Delta m = k \cdot m_{prior} \tag{3}$$

is the arbitrary mass added. This increase in mass causes an increase in the minimal power requirement

$$\Delta P_{req,min} = 4 \cdot \sqrt[4]{\frac{4 \cdot c_{D0}}{27\pi^3 e^3 \Lambda} \frac{(mg)^{\frac{3}{2}}}{b\sqrt{\rho}}} \left([k+1]^{\frac{3}{2}} - 1\right) \tag{4}$$

which, for a solar powered airplane using solar radiation as the only energy source, has to be compensated by adding more solar cells. As we assume proportionality of solar power generated, solar cell area and solar cell mass,

$$P_{solar} \sim A_{solar} \sim m_{solar} \tag{5}$$

the increase in the minimal power requirement also causes an increase in the solar cell mass needed to fulfill the energy demand

$$\Delta m_{solar} = m_{solar,prior} \left([k+1]^{\frac{3}{2}} - 1\right) \tag{6}$$

The prior solar cell mass $m_{solar\_cell,prior}$ can be calculated using

$$m_{solar,prior} = P_{req,min,prior} \cdot f_{solar} \frac{\sigma_{solar}}{\eta_{solar}} \tag{7}$$

whereas η_{solar} is the solar cell efficiency, σ_{solar} is the solar cell specific weight and f_{solar} is the solar cell factor describing, which area in solar cells (given in square meter) are necessary in order to provide $1W$ continuous power, assuming solar cell efficiency $\eta_{solar} = 1$ and storage efficiency $\eta_{store} = 1$. f_{solar} can be calculated from a simple energy balance or be taken from precaluclated charts. Figure 1 shows the solar irradiance for a horizontal solar cell at high altitude [1] and the constant systems energy demand P_{system}. The areas under both curves have to be equal over a 24 hour cycle for an energy balanced system. As $P_{solar} \sim A_{solar}$ one can solve

$$E = \int_{0h}^{24h} I_{solar} \cdot f_{solar}\, dt = \int_{0h}^{24h} P_{system}\, dt \tag{8}$$

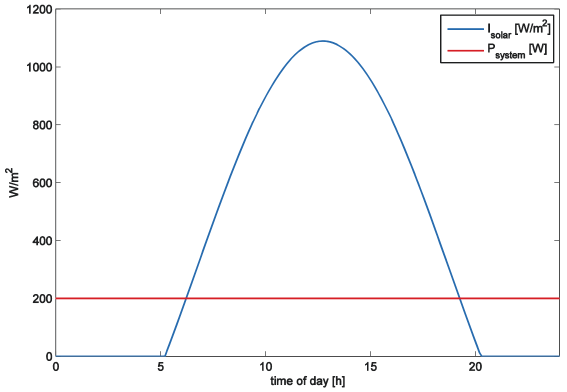

Figure 1: Solar irradiance and system power demand over a 24h cycle, for a horizontal cell on summer solstice and at 23km height.

This also works for tilted or alignable solar cells, therefore I_{solar} can be adjusted using the equations provided in [1] (p.25 ff.), when trying to maximize solar power by keeping the irradiation impact angle at 90°.

Figure 2 shows a precalculated chart with f_{solar} as a function of latitude and for various dates using a horizontal solar cell.

We now assume that $P_{req,min}$ is strictly monotonic increasing with additional mass, which seems justified by equation (1). Hence adding more solar cells in order to compensate the additional power requirement once again causes an increase in mass and creates a loop. It can be seen that this loop can be stable, when $\Delta m_{solar} < k \cdot m_{prior}$, but not necessarily has to be. But for sure it is unstable when $\Delta m_{solar} > k \cdot m_{prior}$, by the assumption of strictly increasing. This leads to the reformulated criterion using the relations above

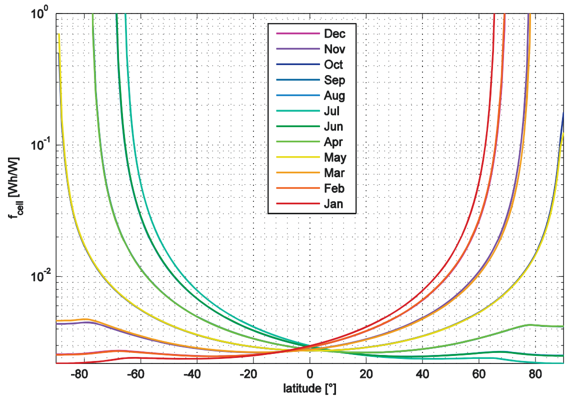

Figure 2: f_{solar_cell} for all months (21st) for a horizontal cell at 23km altitude. The highest value for the corresponding latitude is the limit for infinite flight and therefor to pick for the criterion evaluation.

$$4 \cdot \sqrt[4]{\frac{4 \cdot c_{D0}}{27\pi^3 e^3 \Lambda} \frac{(mg)^{\frac{3}{2}}}{b\sqrt{\rho}}} \left([k+1]^{\frac{3}{2}} - 1\right) f_{solar} \frac{\sigma_{solar}}{\eta_{solar}} < k \cdot m \tag{9}$$

Figure 3 shows the plotted criterion as a function of the solar cell technology $(\eta_{solar}/\sigma_{solar})$ for various increasing factors k. It can be seen, that for small ks the criterion converges towards what seems to be a physical limit. Taking the limit of k towards zero of the formula above

$$\lim_{k\to 0}\left[\frac{\left([k+1]^{\frac{3}{2}}-1\right)}{k}\cdot C_1\right]=\frac{3}{2}\cdot C_1 \qquad (10)$$

leads to the following total mass criterion

$$6\cdot\sqrt[4]{\frac{4\cdot c_{D0}}{27\pi^3 e^3\Lambda}\frac{(mg)^{\frac{3}{2}}}{b\sqrt{\rho}}}\,f_{solar}\frac{\sigma_{solar}}{\eta_{solar}}<m \qquad (11)$$

which cannot be exceeded by any airplane performing infinite flight and using solar power as its only energy source.

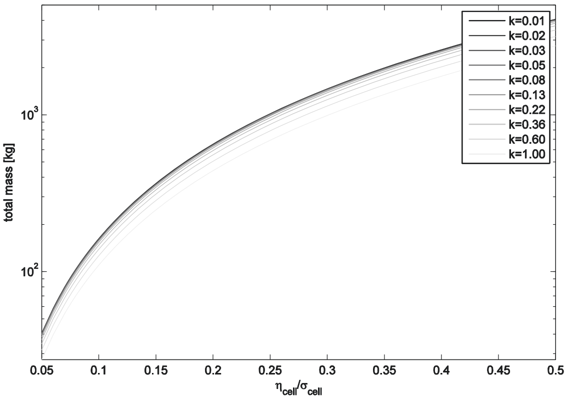

Figure 3: Evaluated criterion [equation (9)] for various increasing factors k. Evaluation parameters: 52°N, e = 0.95, c_{D0} = 0.03, Λ = 16.3, b = 90m, ρ@23km = 0.05486kg/m^3, f_{cell} = 0.0158 (from chart)

B. Storage and Propulsion Considerations

This criterion is very optimistic because a higher power consumption not only increases the solar cell mass, but also the battery mass in order to store the energy used during the day, the motor weight, the gear weight, additional structure mass and so on. Also, in reality $P_{req,min}$ is hardly ever achieved due to additional losses not taken into account by equation (1). One of the largest factors of these is the battery weight. Similarly to what was done with the increase in solar cell weight, can also be done for the battery. We assume

$$m_{store}\sim E_{store}\sim\frac{P_{system}}{\eta_{store}} \qquad (12)$$

so all energy storage is assumed to be done via an onboard system that requires mass and not in altitude. Therefore

$$\Delta m_{store}=P_{system}\cdot f_{store}\cdot\frac{\sigma_{store}}{\eta_{store}} \qquad (13)$$

whereas η_{store} is the storage efficiency defined by $\eta_{store}=P_{battery\_out}/P_{battery\_in}$, σ_{store} is the battery specific weight $[kg/Wh]$ and f_{store} is the storage factor describing, how many Wh of storage are necessary per $1W$ continuous system power requirement, assuming solar cell efficiency $\eta_{solar}=1$ and storage efficiency $\eta_{store}=1$. f_{store} can be calculated similarly to the solar cell factor either by integrating the greater than zero part of $P_{solar}-P_{system}$ or with precalculated charts (figure 4).

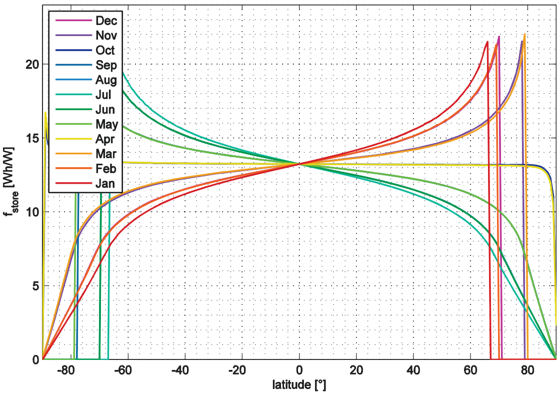

Figure 4: f_{store} for all months (21st) per $1W$ system demand. The highest value for the corresponding latitude is the one limiting for infinite flight and therefor to pick for the criterion evaluation.

Getting back to equation (11), the new criterion is that Δm_{solar}, cause by adding arbitrary weight, not only has to be smaller than the added weight $k\cdot m_{prior}$, but that the sum of all caused mass increased due to the added weight have to be smaller than the added weight itself.

$$\Delta m_{solar}+\Delta m_{store}<k\cdot m_{prior} \qquad (14)$$

Hence the reformulated criterion is the following

$$6\cdot\sqrt[4]{\frac{4\cdot c_{D0}}{27\pi^3 e^3\Lambda}\frac{(mg)^{\frac{3}{2}}}{b\sqrt{\rho}}}\left[f_{solar}\frac{\sigma_{solar}}{\eta_{solar}}+f_{store}\frac{\sigma_{store}}{\eta_{store}}\right]<m \qquad (15)$$

When taking propulsion efficiency effects into account, it can be seen, that the system actually needs more energy than calculated in equation (1)

$$P_{system}=\frac{1}{\eta_t}P_{req,min} \qquad (16)$$

where $\eta_t=\eta_{motor}\cdot\eta_{regulator}\cdot\eta_{gear}\cdot\eta_{propeller}$ is the propulsion efficiency. All weights of equation (15) are related to the true system power requirement, which leads to

$$\frac{1}{\eta_t}\cdot 6\cdot\sqrt[4]{\frac{4\cdot c_{D0}}{27\pi^3 e^3\Lambda}\frac{(mg)^{\frac{3}{2}}}{b\sqrt{\rho}}}\left[f_{solar}\frac{\sigma_{solar}}{\eta_{solar}}+f_{store}\frac{\sigma_{store}}{\eta_{store}}\right]<m \qquad (17)$$

Motor-, gear and regulator weights can be taken into account similarly to the other weights as

$$m_{motor,regulator,grear}\sim P_{system} \qquad (18)$$

Which leads to the final reformulated criterion

$$6\cdot\sqrt[4]{\frac{4\cdot c_{D0}}{27\pi^3 e^3\Lambda}\frac{(mg)^{\frac{3}{2}}}{b\sqrt{\rho}}}\left[f_{solar}\frac{\sigma_{solar}}{\eta_t\cdot\eta_{solar}}+f_{store}\frac{\sigma_{store}}{\eta_t\cdot\eta_{store}}+\frac{(\sigma_{motor}+\sigma_{reg}+\sigma_{gear})}{\eta_t}\right]<m \qquad (19)$$

This formula presents a simple way of calculating a maximal system weight of airplanes, designed for infinite flight using solar power only. It's a function of the sum of multiple specific weight to efficiency ratios σ/η of various subsystems, where the propulsive efficiency η_t has the largest influence.

III. APPLICATION

Equation (19) presents a simple and fast way of estimating the feasibility of a solar HALE project using current solar cell, propulsion and battery technology, e.g. answering the question: Is it possible to have a stationary solar UAV for infinite flight at 23km altitude using the technical data described below figure 5 and weighing 3 tons? The criterion tells us that, when only using the solar model, for $\eta_{solar}/\sigma_{solar}$ below 0.45, it is not. And when taking the other components into account it won't be for values much above.

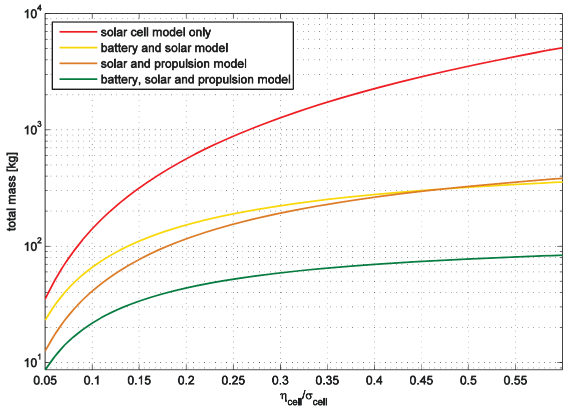

Figure 5: Evaluated criterion for the different model details. Evaluation parameters: 52°N, $e = 0.95$, $c_{D0} = 0.03$, $\Lambda = 16.3$, $b = 90m$, $\rho@23km = 0.05486kg/m^3$, $f_{cell} = 0.017$ (from chart), $f_{store} = 16.97$ (from chart), $\eta_{store} = 0.98$, $\sigma_{store} = 1/220kg/Wh$, $\eta_t = 0.67$, $\sigma_t = \sigma_{motor} + \sigma_{regulator} + \sigma_{gear} = 0.061kg/W$

A. Validation

In order to validate the criterion derived in section II, two solar HALE designs for infinite flight were tested against it. First, SkySailor [4] with the following technical data was examined:

c_{D0}	0.006	σ_{store}	$0.00526\frac{kg}{Wh}$
e	0.9	η_{store}	95%
Λ	12.9	η_t	66%
b	$3.2m$	σ_t	$0.008\frac{kg}{W}$
σ_{solar}	$0.581\frac{kg}{m^2}$	latitude	47°N (Zurich)
η_{solar}	16.9%	altitude	500m (design pt.)

Table 1: SkySailor technical specifications [4]

Figure 6 shows the criterion for SkySailor at design altitude of $500m$ above sea level, with a total weight of $2.55kg$ (dash-dotted black line), SkySailor fulfills all the different criterion models. Maximum weight for those parameters would lie at about $4.93kg$.

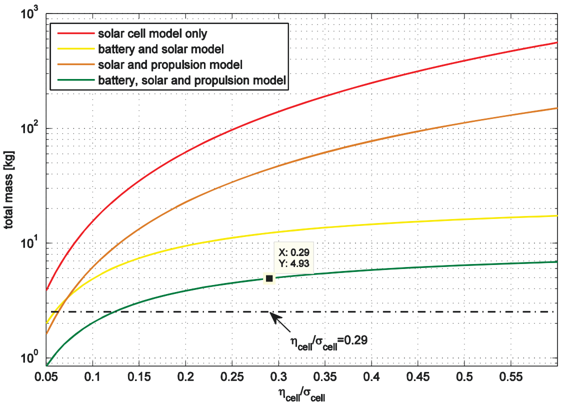

Figure 6: Evaluated criterion for SkySailor at design point.

However, at an altitude of 20km, SkySailor would not be able to perform infinite flight without storing energy in gaining altitude. The solar and the propulsion criterion are fulfilled, but not the battery model assuming all energy is stored in mass consuming batteries or fuel cells and not in altitude.

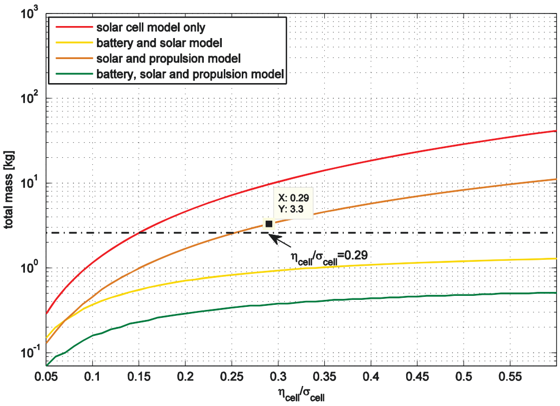

Figure 7: Evaluated criterion for SkySailor at 20km altitude. Dash-dotted line is the real systems weight.

Second, NASAs Helios HP01 with the following given specifications was examined:

c_{D0}	$(0.005 - 0.01)$	σ_{store}	$\left(0.005 - 0.002\frac{kg}{Wh}\right)$
e	$(0.85 - 0.95)$	η_{store}	(95% – 60%)
Λ	30.9	η_t	(60% – 70%)
b	$75.3m$	σ_t	$\left(0.01 - 0.03\frac{kg}{W}\right)$
σ_{solar}	$\left(0.6 - 2.0\frac{kg}{m^2}\right)$	latitude	21°N (Hawaii)
η_{solar}	19%	altitude	23km (design pt.)

Table 2: Helios HP01 technical specifications [9]

Due to the limited availability of to the Helios HP01 prototype specifications, as well as for most other systems, the values in brackets had to be estimated. This is done using a best and worst case scenario, including two different designs for energy storage, LiPo-Batteries with an energy density of $5\frac{kg}{Wkh}$ and an charge discharge efficiency of 95%, and fuel cells with an energy density of $2\frac{kg}{Wkh}$ and an charge discharge efficiency of 60%. Hence the criterion defines not single lines but feasibility regions, depending on how good the unknown quantities are said to be.

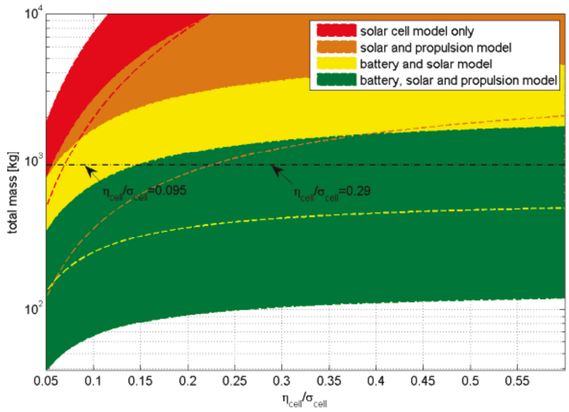

Figure 8: Evaluated criterion for Helios HP01 at design point. Feasibility regions due to estimated values, dashed lines are hidden, lower boundaries; dash-dotted line is the real systems weight.

Figure 8 shows the feasibility regions for the Helios HP01 design using the data given in table 2. The white region on the lower side is feasible for all models and estimates. The different areas show the different models having the most pessimistic estimate as the lower border and the most optimistic as the upper. When regions overlap, the lower border is marked with dashed lines. The Helios HP01 design has a total weight of $929kg$ (dash-dotted black line) and hence fulfills for the best case specifications ($\eta_{solar}/\sigma_{solar}$ = 0.29) all the models (optimistic estimate = below all the upper borders). Taking the worst case values ($\eta_{solar}/\sigma_{solar}$ = 0.095) it merely fulfills the solar cell criterion, but not the others (pessimistic estimate = below the lower borders). One can hereby say, the systems capability for infinite, stationary flight would be plausible when specifications are close to the optimistic estimate.

IV. CONCLUSION

In this paper, a simple and modular formula to calculate the limit of total system weight for infinite solar flight was derived. Depending on what to take into account, the models include solar-cell, propulsion and energy storage considerations. As the criterion was derived from a mass-energy-perspective, it is not limited to a specific size of the airplane or cruising altitude, which was shown in the validation section with two very different systems. This criterion could be used in order to check the feasibility of existing systems as well as ones in the state of concept and serve as a break the loop criterion in an iterative design process, when the mass limit was exceeded.

A. Future Work

The presented method relies on energy storage using mass consuming devices such as batteries or fuel cells. When the airplane leaves the assumption for level flight and begins storing energy in altitude, one has to leave the energy storage model aside. More investigation into variable altitude energy storage and performance evaluation would be interesting.

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