INSTITUT FÜR LUFT- UND RAUMFAHRT
FACHGEBIET FLUGMECHANIK, FLUGREGELUNG, UND AEROELASTIZITÄT

Trajectory Optimization with Dynamic Programming

WORKSHOP MISSIONSFÜHRUNG, BANHFÜHRUNG UND BAHNPLANUNG
FÜR INNOVATIVE LUFTFAHRTANWENDUNGEN
24TH MAY 2023

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Motivation

Subject of talk

DP →

```
1 0.917696913225422 0.242881414970932 600 27.5 1 2 0
2 0.919265348838039 0.242881414970932 1100 27.5 1 2 0.0499583957219428
3 0.922402220063275 0.242881414970932 700 30 1 2 -0.0199973339731505
4 0.923970655675893 0.242881414970932 1200 27.5 1 2 0.0499583957219428
5 0.928675962513747 0.242881414970932 600 30 1 2 -0.0199973339731505
6 0.930244398126365 0.242881414970932 1100 27.5 1 2 0.0499583957219428
7 0.9333812693516 0.242881414970932 700 30 1 2 -0.0199973339731505
8 0.934949704964218 0.242881414970932 1200 27.5 1 2 0.0499583957219428
9 0.939655011802072 0.242881414970932 600 30 1 2 -0.0199973339731505
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12 0.945928754252543 0.242881414970932 1200 27.5 1 2 0.0499583957219428
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14 0.952202496703015 0.242881414970932 1100 27.5 1 2 0.0499583957219428
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17 0.961613110378722 0.242881414970932 600 30 1 2 -0.0199973339731505
18 0.96318154599134 0.242881414970932 1100 27.5 1 2 0.0499583957219428
19 0.966318417216576 0.242881414970932 700 30 1 2 -0.0199973339731505
20 0.967886852829194 0.242881414970932 1200 27.5 1 2 0.0499583957219428
21 0.972592159667047 0.242881414970932 600 30 1 2 -0.0199973339731505
22 0.972592159667047 0.242881414970932 600 30 1 2 -0.0199973339731505
23 0.972592159667047 0.242881414970932 600 30 1 2 -0.0199973339731505
24 0.972592159667047 0.242881414970932 600 30 1 2 -0.0199973339731505
25 0.972592159667047 0.242881414970932 600 30 1 2 -0.0199973339731505
26 0.972592159667047 0.242881414970932 600 30 1 2 -0.0199973339731505
27 0.972592159667047 0.242881414970932 600 30 1 2 -0.0199973339731505
28 0.972592159667047 0.242881414970932 600 30 1 2 -0.0199973339731505
29 0.972592159667047 0.242881414970932 600 30 1 2 -0.0199973339731505
30 0.972592159667047 0.242881414970932 600 30 1 2 -0.0199973339731505
31 0.972592159667047 0.242881414970932 600 30 1 2 -0.0199973339731505
32 0.972592159667047 0.242881414970932 600 30 1 2 -0.0199973339731505
```

Flight Plan →

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Increased Efficiency

Reduced energy consumption w/ feasible trajectories

Longitudinal flight navigation and guidance, focus on optimal flight path angles (FPA) and speeds for battery-electric aircraft

Objective

Build onboard trajectory generator for electric glider aircraft

- Fast convergence
- Consider limits of powertrain, weather and terrain

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Chose Dynamic Programming (DP) approach

Dynamic Programming applied to Aircraft

Harada et al. [7] : piecewise linear approximation for aircraft longitudinal flight path optimization, uses BADA consumption model

Oettershagen et al.[8]: Inclusion of weather data, lateral navigation with DP, using SOC with a battery efficiency parameter

Ahmed et al. [9]: ten time more coarse grid in FPA, using BADA consumption model (fuel flow), MDP-approach, basis of the approach used in this presentation

- List not exhaustive

Why Dynamic Programming?

Other Methods:

- Direct numerical solutions (e.g. collocation)
- Analytical/indirect (e.g. maximum principle)

require good initialization of the states and controls, for indirect case also the associated costates [5]

- Global ‚optimal' solution (highly depends on gridding)
- Predictable run-time of algorithm
- DP finds global solutions <u>without initial guess, no gradients needed</u>

Bellman

Bellman: Working with multi-stage decision processes

Bellman in his book [6]*:* *To Betty-Jo* (his wife) :
whose decision processes defy analysis

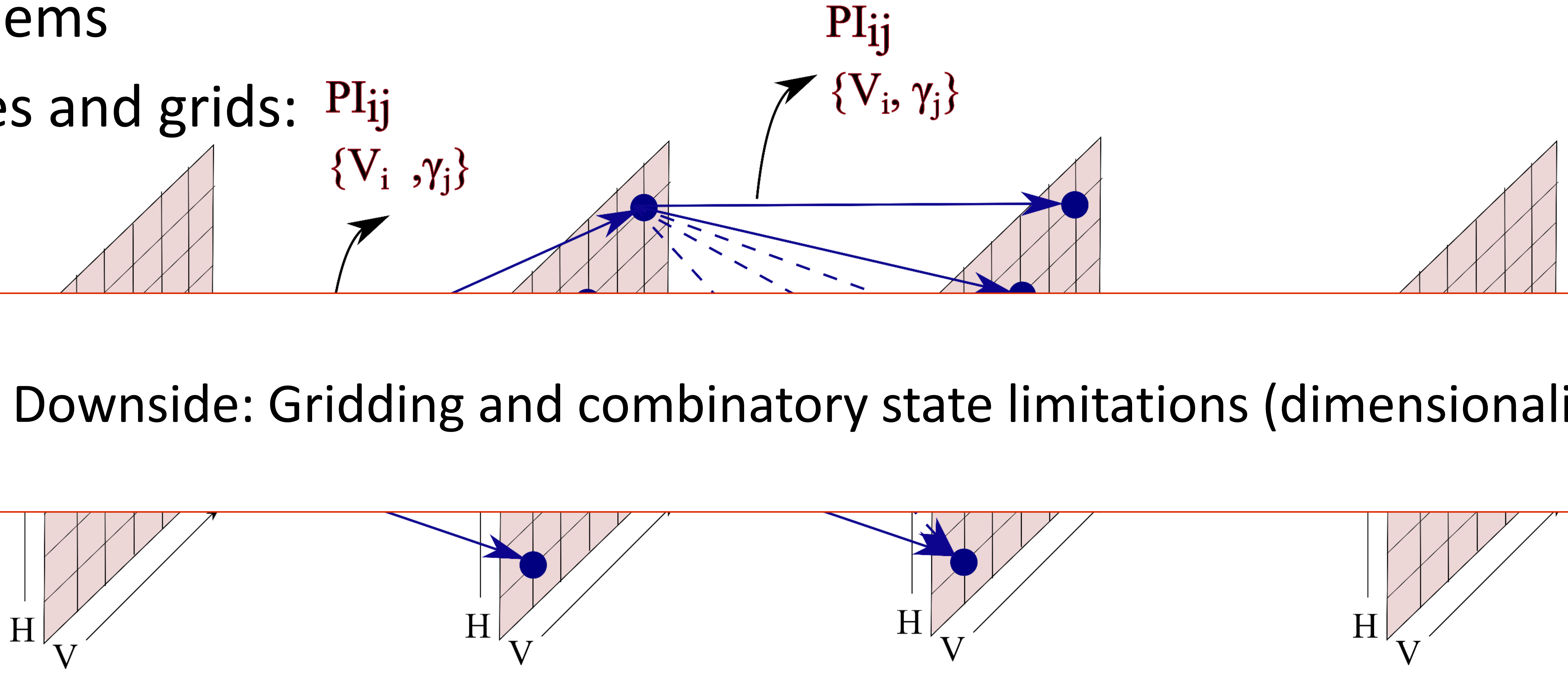
Bellman*: PRINCIPLE OF OPTIMALITY. An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.* → *Decision process can be split in subproblems*

$$J = \int_{t_0}^{t_1} \mathscr{L}(\boldsymbol{x},\boldsymbol{u},t)dt + \int_{t_1}^{t_f} \mathscr{L}(\boldsymbol{x},\boldsymbol{u},t)dt$$

What is Dynamic Programming?

Application of *PRINCIPLE OF OPTIMALITY:*

- Button-up approach: Make use of memoized results for recurring sub-problems
- Stages and grids:



Downside: Gridding and combinatory state limitations (dimensionality)

Application

Gridding – Select finite set of states and inputs for analysis of model and controller performance

Database – Use model and controller to generate performance database (PIs) on grid

Most time consuming → offline

Reachability – Create logical matrices indicating state trainsitions the controller can fly

Search Space – Only flyable and transitions will be evaluated in the DP algorithm

Speed up algorithm

Algorithm – DP algorithm executes using the performance database, search spaces and interpolation

algorithm for onboard application

Flight Plan – Output of the DO algorithm is the optimal control history which is written into a flight plan

Optimized Trajectory

Presentation Roadmap

Next steps in this presentation:

- Generate database of aircraft performance
- Run dynamic programming that includes effect of the current on the SOC changes (Peukert-Effect), terrain and wind
- Compare to approximations
- Compare to direct collocation optimization with internal battery states (Battery Model of Ref [10])

Generate the Database: Models

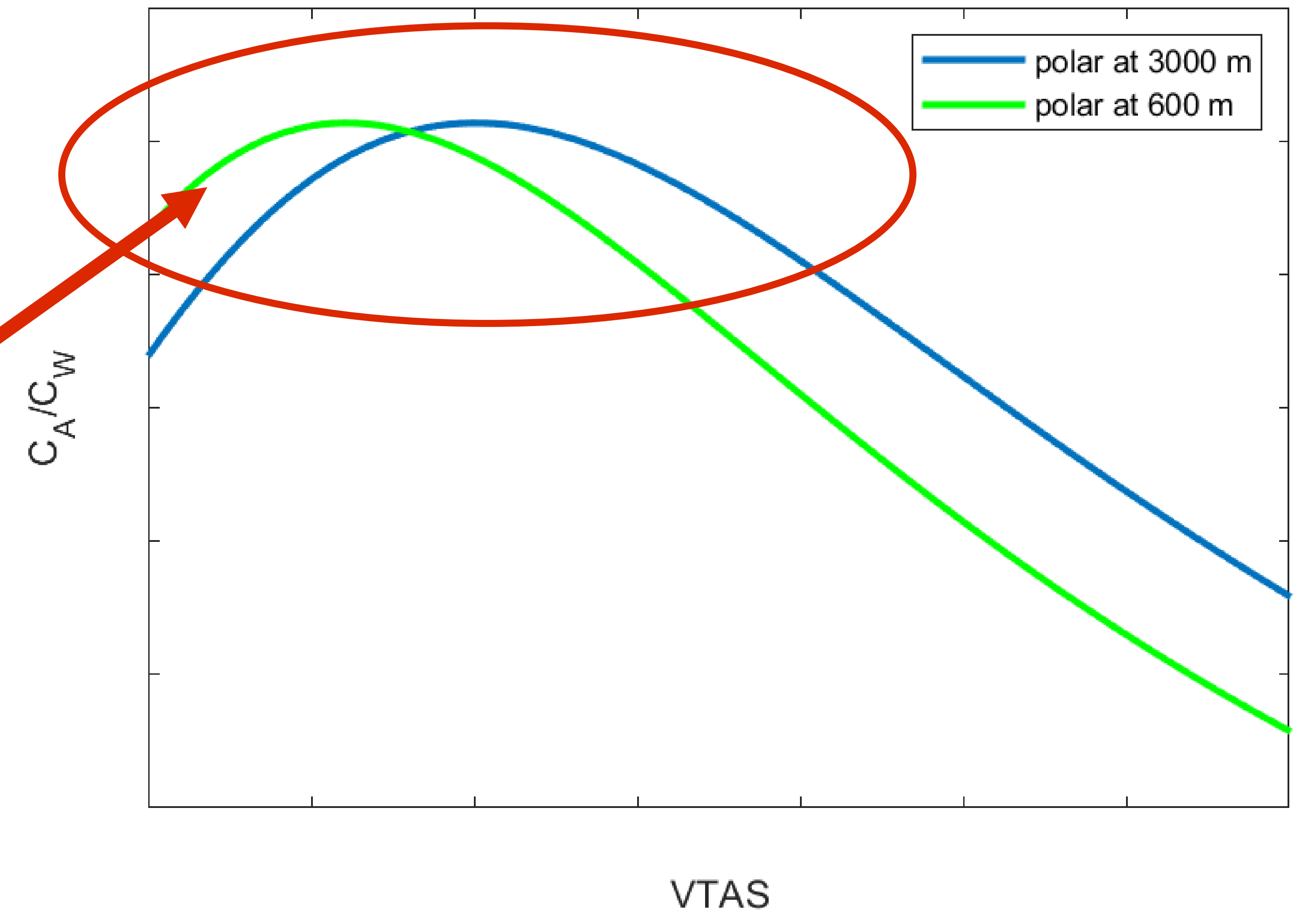
Point mass model including TECS control

States *[x, y, z (h), VTAS, PFA, track, Energy]*

Inputs *[FPA_cmd, VCAS _cmd]*

- EOM include wind in all three axes
- Quadratic drag polar

→ Wind will influence best range speed
→considered later



No factory data!

Generate the Database: Propulsion System Model

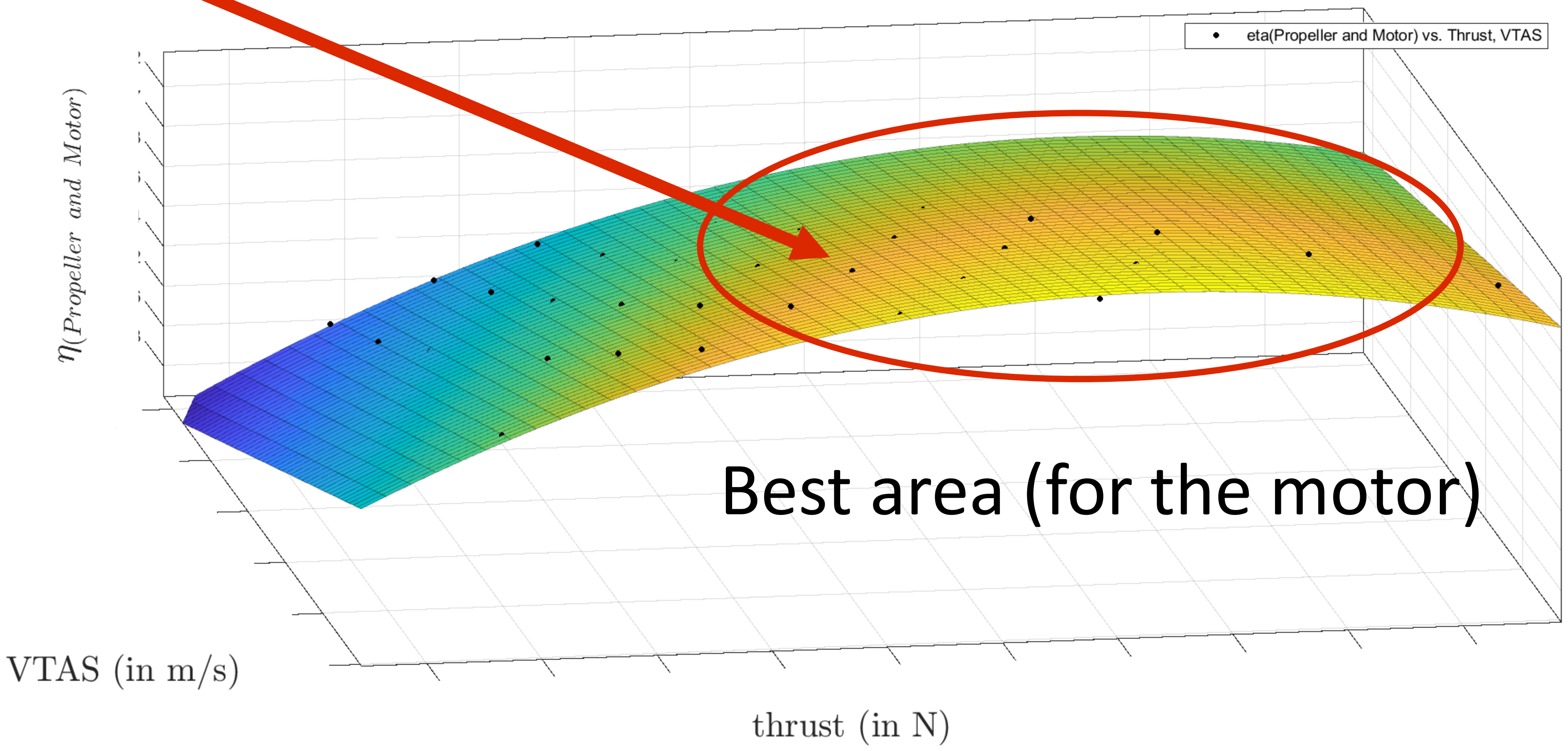
$$P_{el} = \frac{T\,V}{\eta_{(Motor\ +Prop)}}$$

Motor current $I = \frac{P_{el}}{V\_bat}$

Battery discharge - Peukert Effect:

$$I_{eff} = I\left(\frac{I}{I_{nom}}\right)^{(nb\ -1)}$$

$$\text{PI} = \frac{\int P_{el}\frac{I_{eff}}{I}}{\Delta x} \text{ in kWh}$$



No factory data!

Generate the Database: Gridding

ALTITUDE STEPS [M]	600, 650, …, 3000
FLIGHT PATH ANGLE (FPA) STEPS	36 [FROM -4.29 DEG TO 5.71 DEG]
Δx [M]	10.000
VCAS SETPOINTS [M/S]	25, 27.5, 30, 32.5, 35, 40, 50, 60
WINDU [M/S], ZONAL WIND COMPONENT	-30, -20, …, 30
WINDV [M/S], MERIDIONAL WIND COMPONENT	-30, -20, …, 30
WINDW [M/S], VERTICAL WIND COMPONENT	-0.2, -0.1, …, 0.2
WIND DATA LEVELS (WEATHER MODEL) [M]. MODEL OF REFERENCE [1] .	562, 654, 752, 856, 924, 1078, 1196, 1320, 1450, 1584, 1724, 1869, 2020, 2176, 2337, 2505, 2679, 2858, 3044

VCAS setpoints: More advantageous for controller implementation then VTAS

Run dynamic programming - What is Expected?

$$V_{stf,glide} = a\,V_h - \frac{\sqrt{a(aV_h^2 + bV_h + c + Wm - St}}{a}$$, a, b, c from polar fit [2],

γ will result from polar (not controlled in case of gliding flight)

$$PI_{climb} = max(\frac{(1 + \gamma\ E)\ V_{GS,climb}}{I_{Bat,climb}})$$, [3]

Find VCAS and FPA that maximize this criterion (gridding or numerical solution)

Generally: Adverse wind conditions increase speed to fly, advantageous decrease it

Because of the sailplane configuration we expect a sawtooth like profile [4]

Expectation: Static Speed to Fly (S2F) Operating Points at 600 m

Example Gridding for Best Range

VH	S2F (CLIMB, M/S)	FPA (CLIMB, DEG)
-30	60	5.43
-20	50	4.86
-10	35	3.72
0	27.5	2.86
10	25	2.86
20	25	2.58
30	25	2.58

Wind from front (VH -30 to -10)

Wind from back (VH 10 to 30)

Voltage @ 400V (not a variable for DP)

VH	S2F (GLIDE, M/S)
-30	57.90
-20	48.49
-10	39.38
0	30.86
10	23.56
20	18.93
30	19.01

Stall (VH 10 to 30)

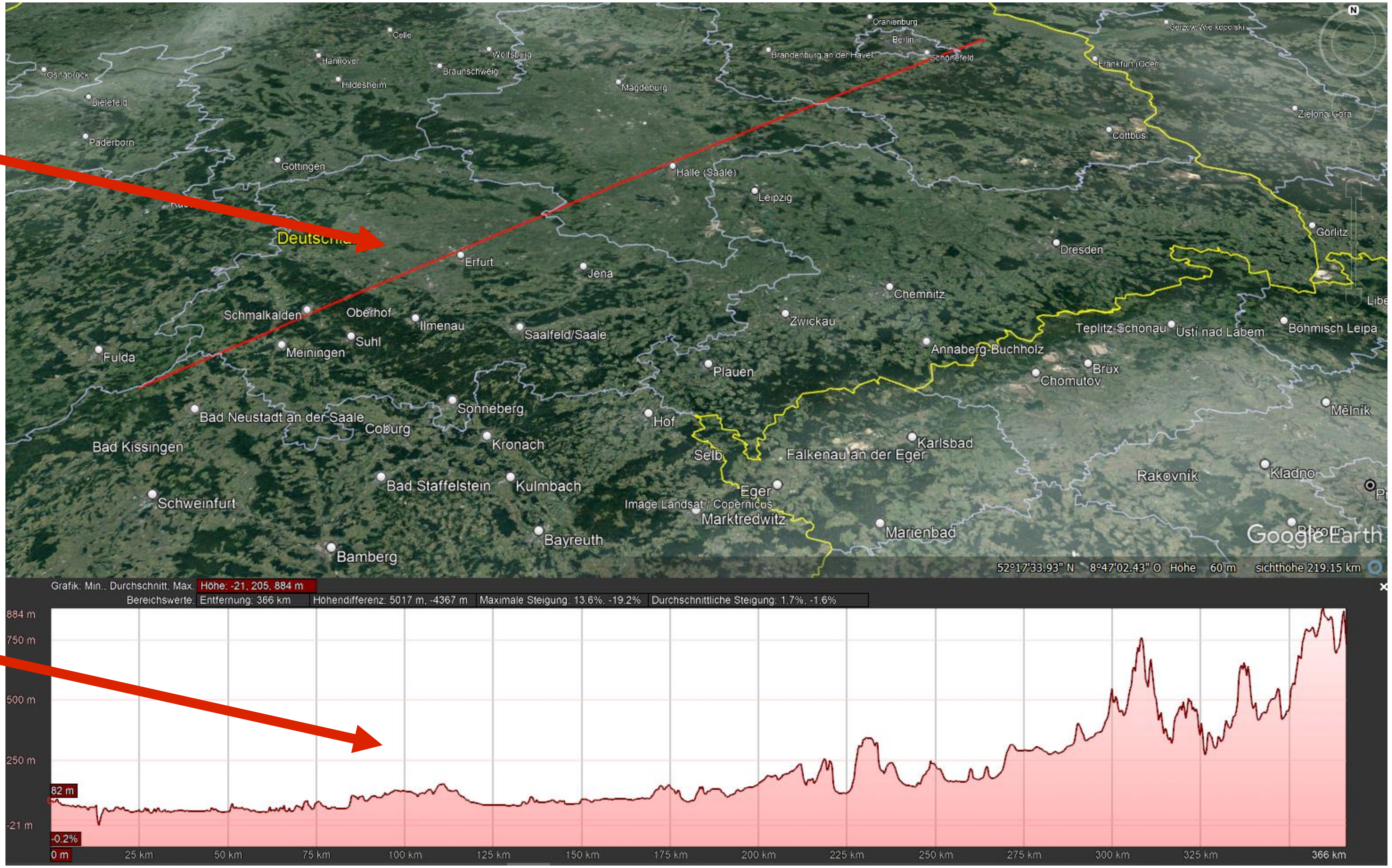
Run dynamic programming

- Generate list of WGS84 coordinates from start to goal
- Load precalculated aircraft performance data and terrain database
- Load wind
- Run DP
- (Write flight plan)

Run dynamic programming: Mission

Orthodrome

Elevation Profile



Run dynamic programming: Flight Task

Strausberg

$H_{ini} = 700\,m\ (\sim 600\,m\ AGL)$

$VCAS_{ini} = 30\ \frac{m}{s}$

ϕ_{ini} = 52.569167 deg

λ_{ini} = 13.918611 deg

Wasserkuppe

$H_{end} = 1600\,m\ (\sim 600\,m\ AGL)$

$VCAS_{end} = 30\ \frac{m}{s}$

ϕ_{end} = 50.492164698 deg

λ_{end} = 9.935996256 deg

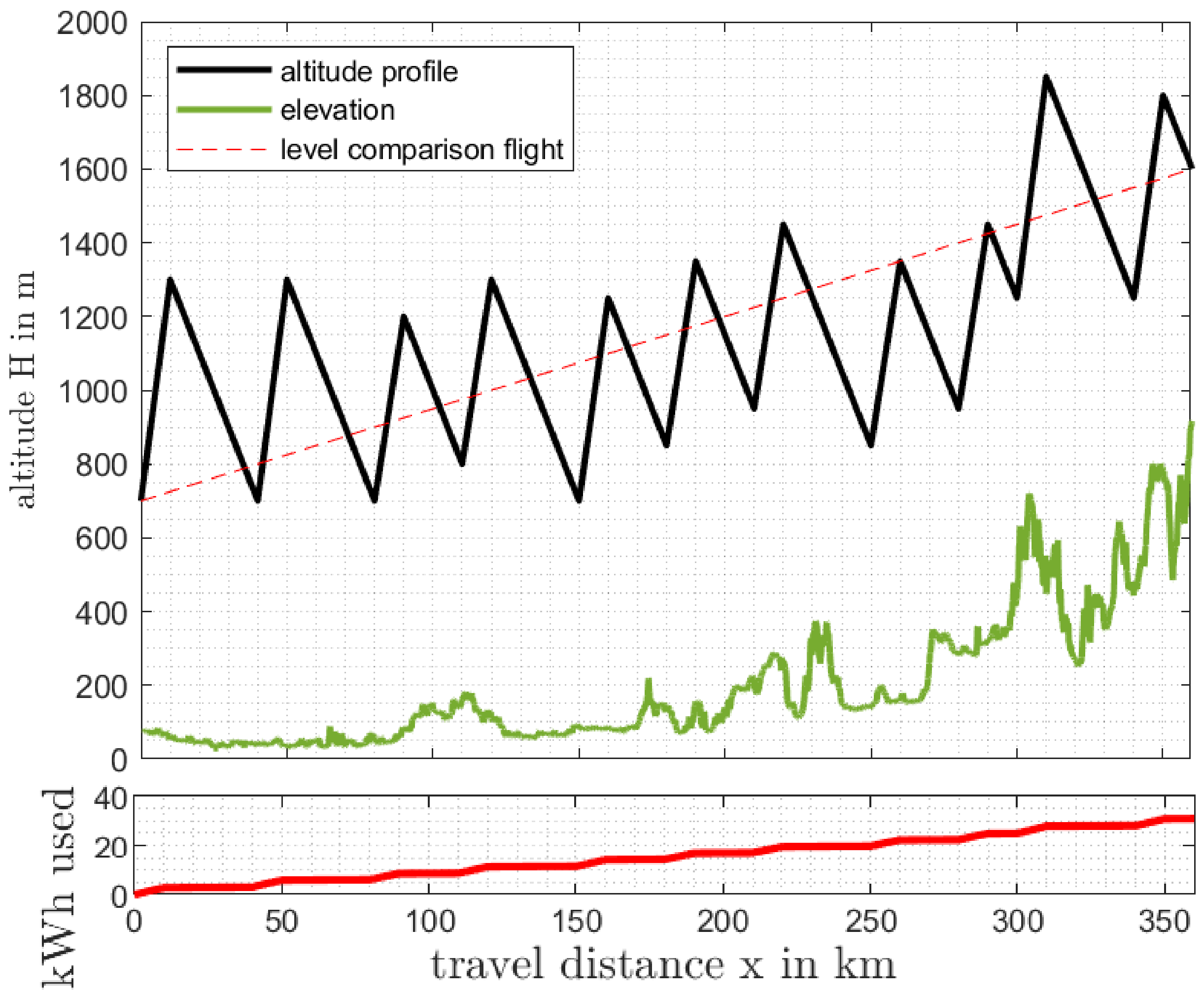
$$\min(PI((H_{ini}, VCAS_{ini}) \dashrightarrow (H_{end}, VCAS_{end})))$$

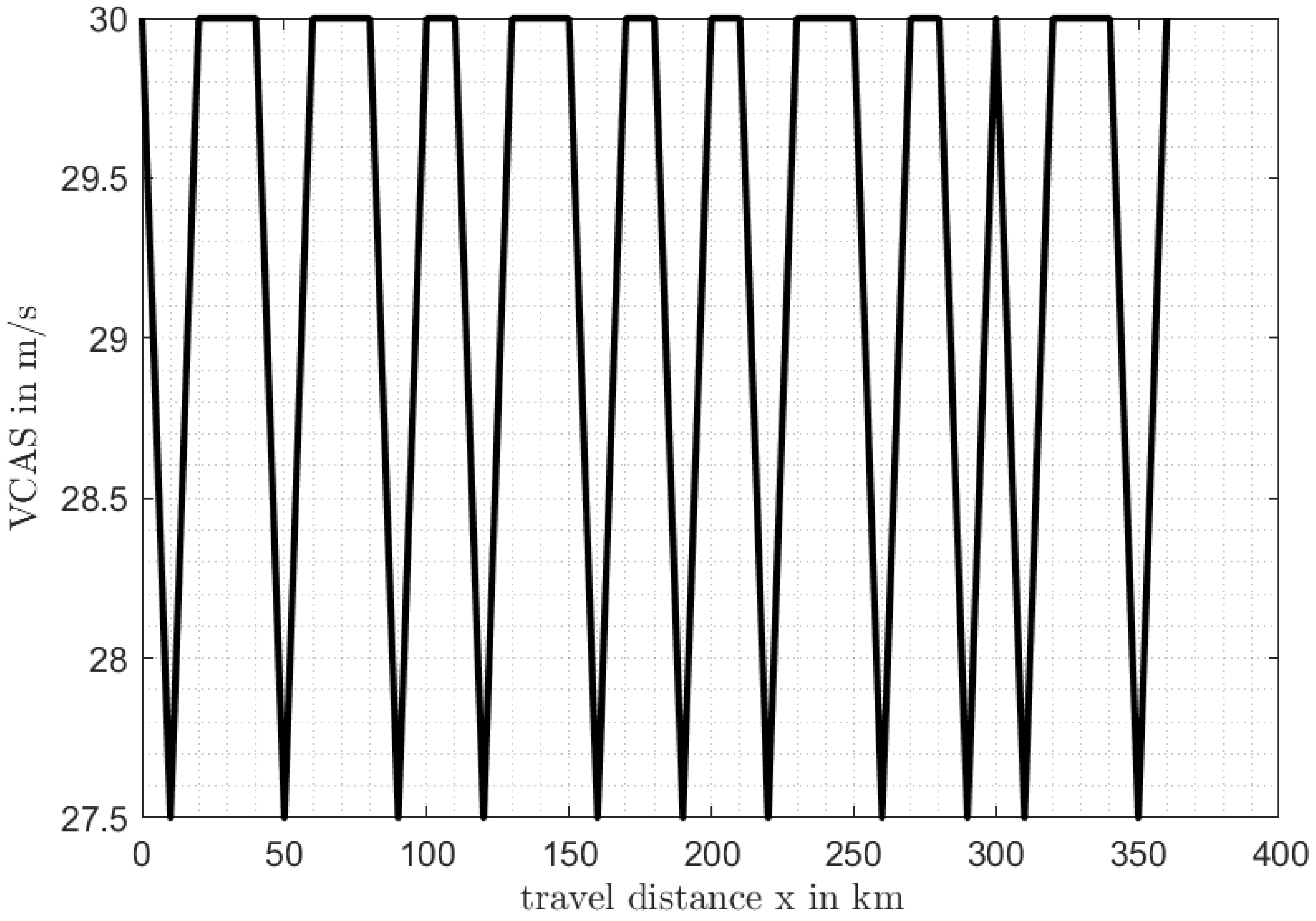
$\mathrm{PI} = \int \frac{P_{el}}{\Delta \mathrm{x}} dx,\quad P_{el}$ weighted with Peukert-Effekt

As a comparsion, level flight consumptions with zero wind best range speed are calculated

Run dynamic programming: Optimization Results w/o Wind

Consumed Energy in optimization: 30.8424 kWh (level: 31.8826 kWh)



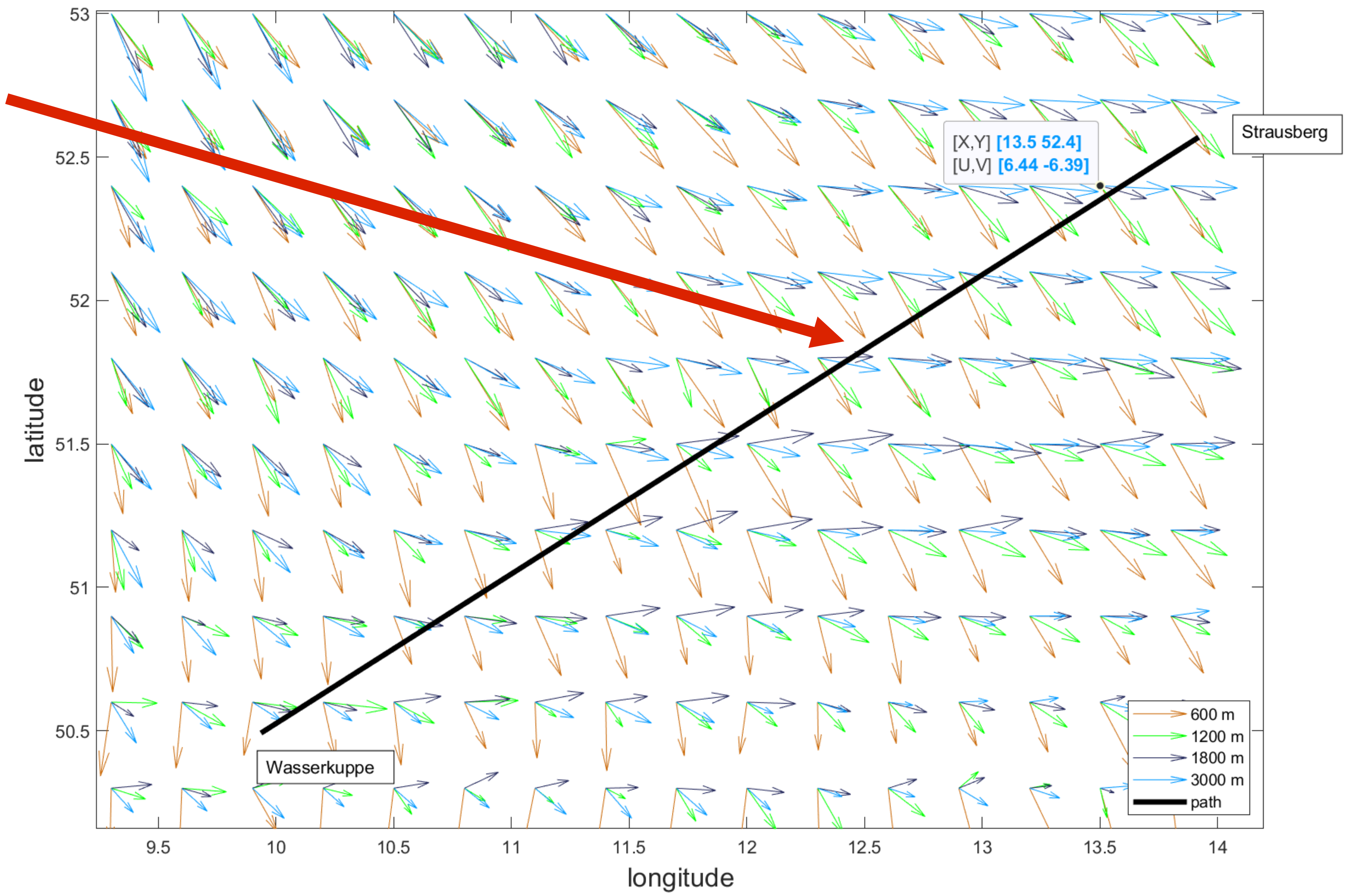


Flying as level as possible from 700 to 1600 m gives 31.8826 kWh

- more than 3 percent advantage with sawtooth/DP

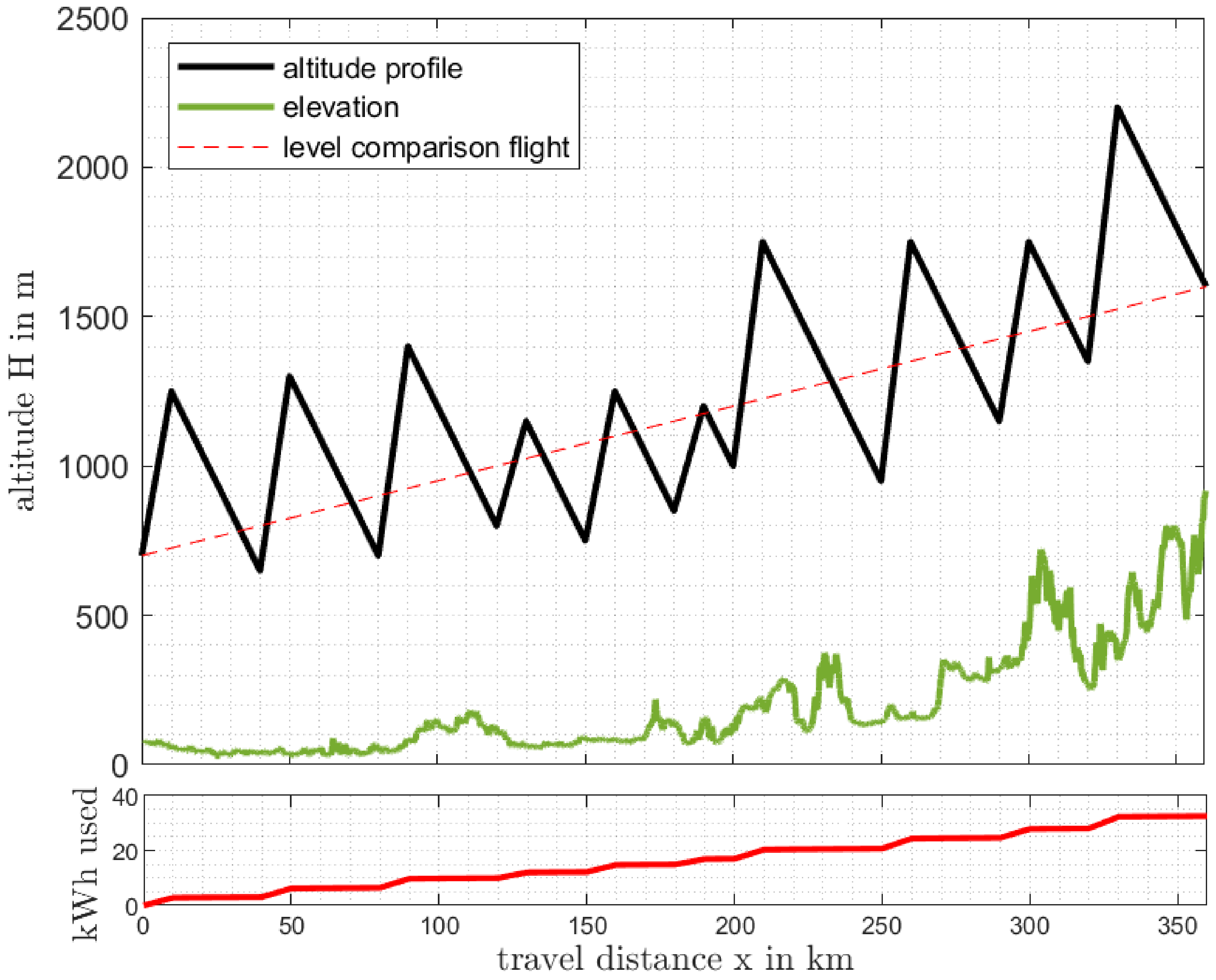
Wind at 02.05.2023, 1800 hours, taken from ICON D2 [1]

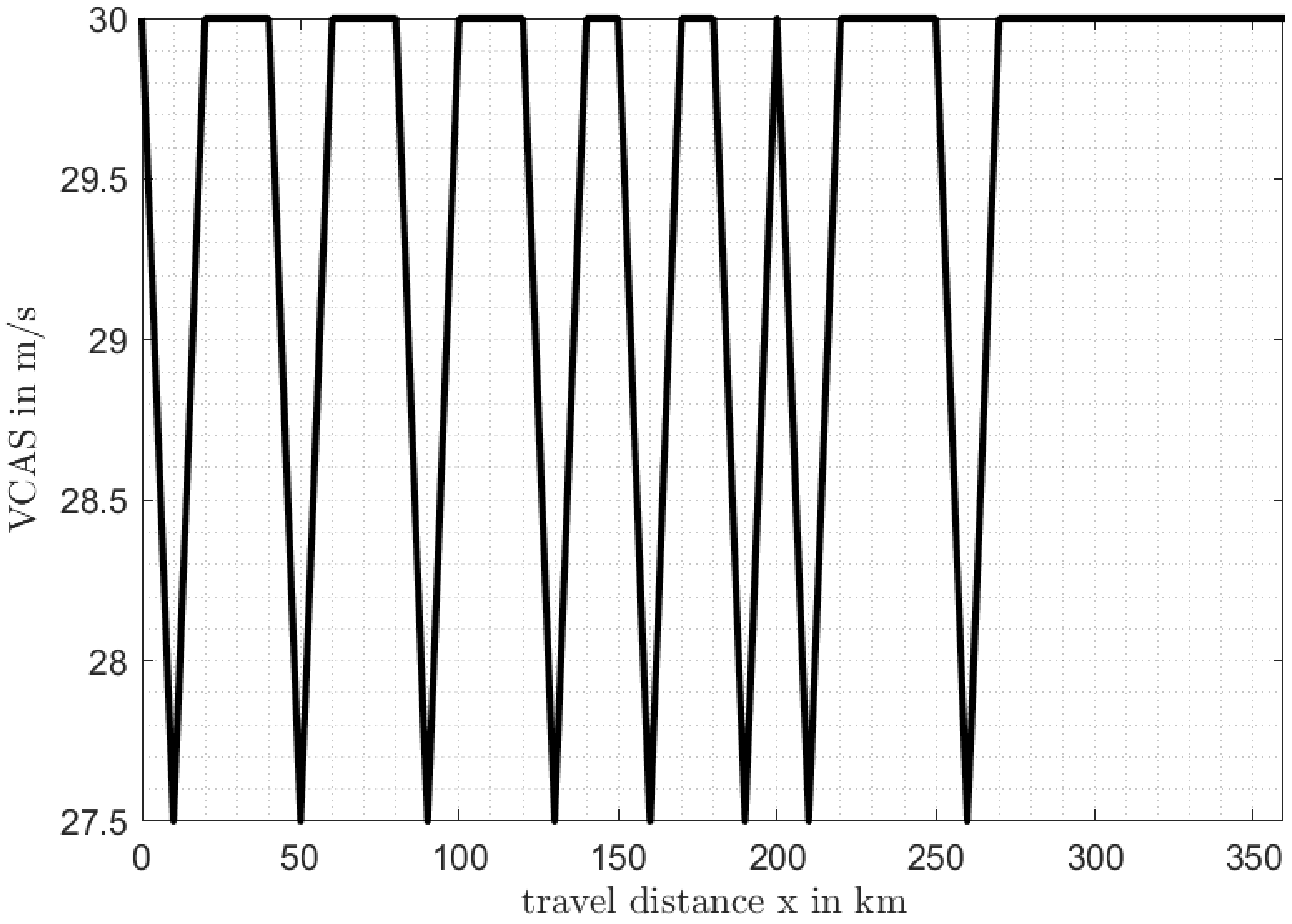
Orthodrome



Run dynamic programming: Optimization Results w/ Wind

Consumed Energy in optimization: 32.5163 kWh (level: approx 37.13 kWh)





Run dynamic programming: Output and Usage of Flight Plan

ID	LAT	LON	ALT	VCAS			FPA
1	0.917696913225422	0.242881414970932	600	27.5	1	2	0
2	0.919265348838039	0.242881414970932	1100	27.5	1	2	0.0499583957219428
3	0.922402220063275	0.242881414970932	700	30	1	2	-0.0199973339731505
4	0.923970655675893	0.242881414970932	1200	27.5	1	2	0.0499583957219428
5	0.928675962513747	0.242881414970932	600	30	1	2	-0.0199973339731505
6	0.930244398126365	0.242881414970932	1100	27.5	1	2	0.0499583957219428
7	0.9333812693516	0.242881414970932	700	30	1	2	-0.0199973339731505
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12	0.945928754252543	0.242881414970932	1200	27.5	1	2	0.0499583957219428
13	0.950634061090397	0.242881414970932	600	30	1	2	-0.0199973339731505
14	0.952202496703015	0.242881414970932	1100	27.5	1	2	0.0499583957219428
15	0.955339367928251	0.242881414970932	700	30	1	2	-0.0199973339731505
16	0.956907803540869	0.242881414970932	1200	27.5	1	2	0.0499583957219428
17	0.961613110378722	0.242881414970932	600	30	1	2	-0.0199973339731505
18	0.96318154599134	0.242881414970932	1100	27.5	1	2	0.0499583957219428
19	0.966318417216576	0.242881414970932	700	30	1	2	-0.0199973339731505
20	0.967886852829194	0.242881414970932	1200	27.5	1	2	0.0499583957219428
21	0.97259215967047	0.242881414970932	600	30	1	2	-0.0199973339731505

Altitudes, flight path angles, VCAS and WGS84 coordinates are written to flight plan.

In segments with negative flight path angle the motor is retracted.

Logic to couple the resulting nonlinear model response to the flight plan setpoints is incorporated. (E.g. do not fly lower than the given EOD altitudes)

Compare to approximations

- R_max = E_Bat/(m*g/(glide ratio*overall efficiency)) [3]
- Calculating the flight task (360 km) consumption with this formula: 30.9073 kWh (assuming 60 % eff.)

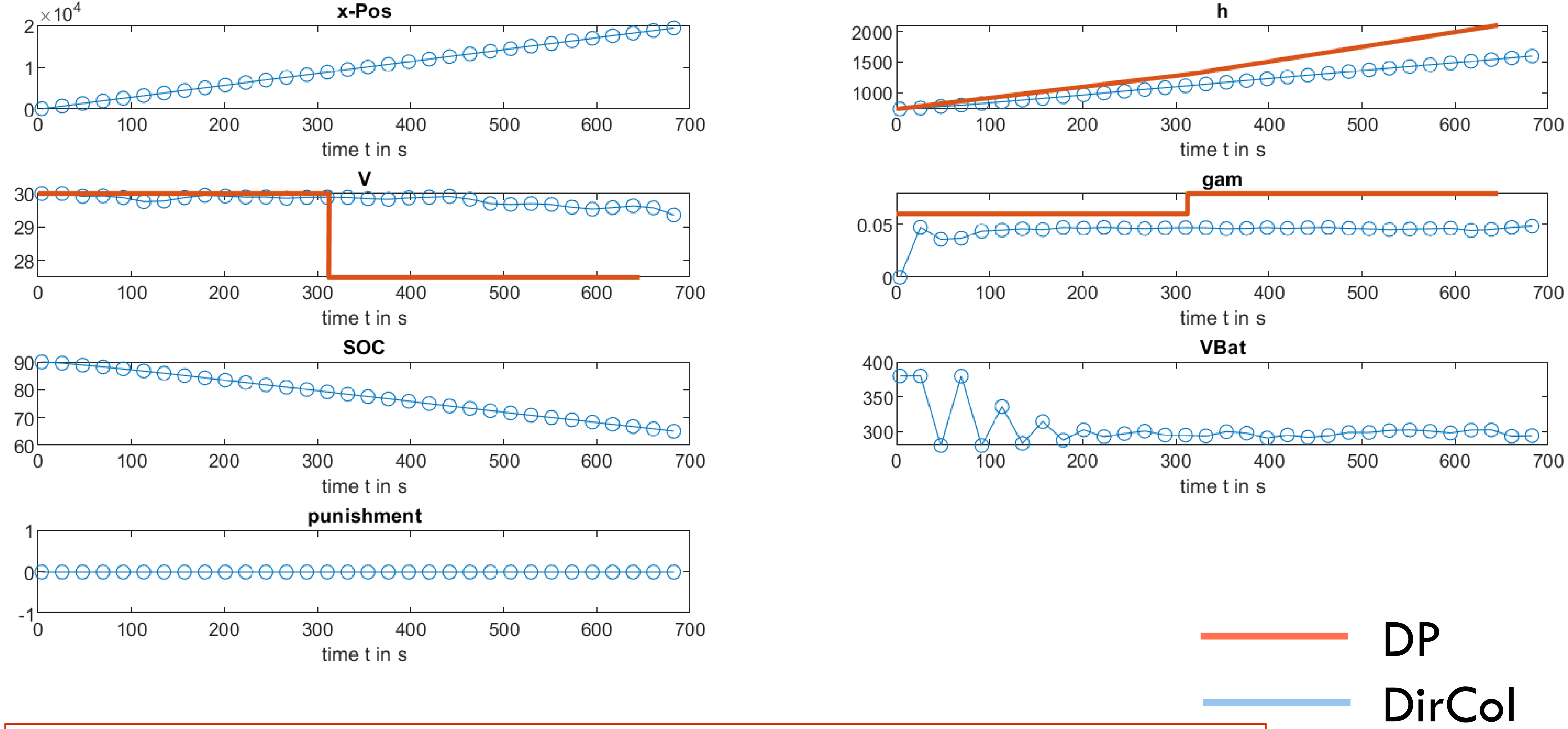
  → fits results (w/o wind) very well
- Peukert-Effect increased consumption by approximately 9.5 %
- Selecting gridding in x to 4 km instead of 10, consumption is only influenced marginally.

Compare to direct collocation optimization

- DirCol is a workhorse of trajectory optimization

- DP Flight Plan warm starts direct collocation algorithm including state of charge and battery voltage

Compare to direct collocation optimization



Battery voltage is the limiting factor with current model

Main Observations

1) Gridding in flight path angle is sufficient
2) Optimum altitudes are plausible with drag and motor model
3) Wind relation to speed-to-fly theory is good, no lateral gridding but small planning areas decrease weather variety and potential benefit of lateral gridding
4) Battery is limiting factor in trajectory

5) Computation time -> MATLAB code 750 s,
   .mex-Function: 2-10 s, depending on granularity of wind lookup in altitude

Next Steps

1) Do in-flight route-replanning with DP and test with nonlinear model
2) Couple this global method to a local optimal flight tactic
3) Might include precipitation and storm risk in PI
4) For inclusion of battery states, battery model has to be tested and validated

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Thank you!

Gefördert durch:

Bundesministerium
für Wirtschaft
und Klimaschutz

aufgrund eines Beschlusses
des Deutschen Bundestages

References

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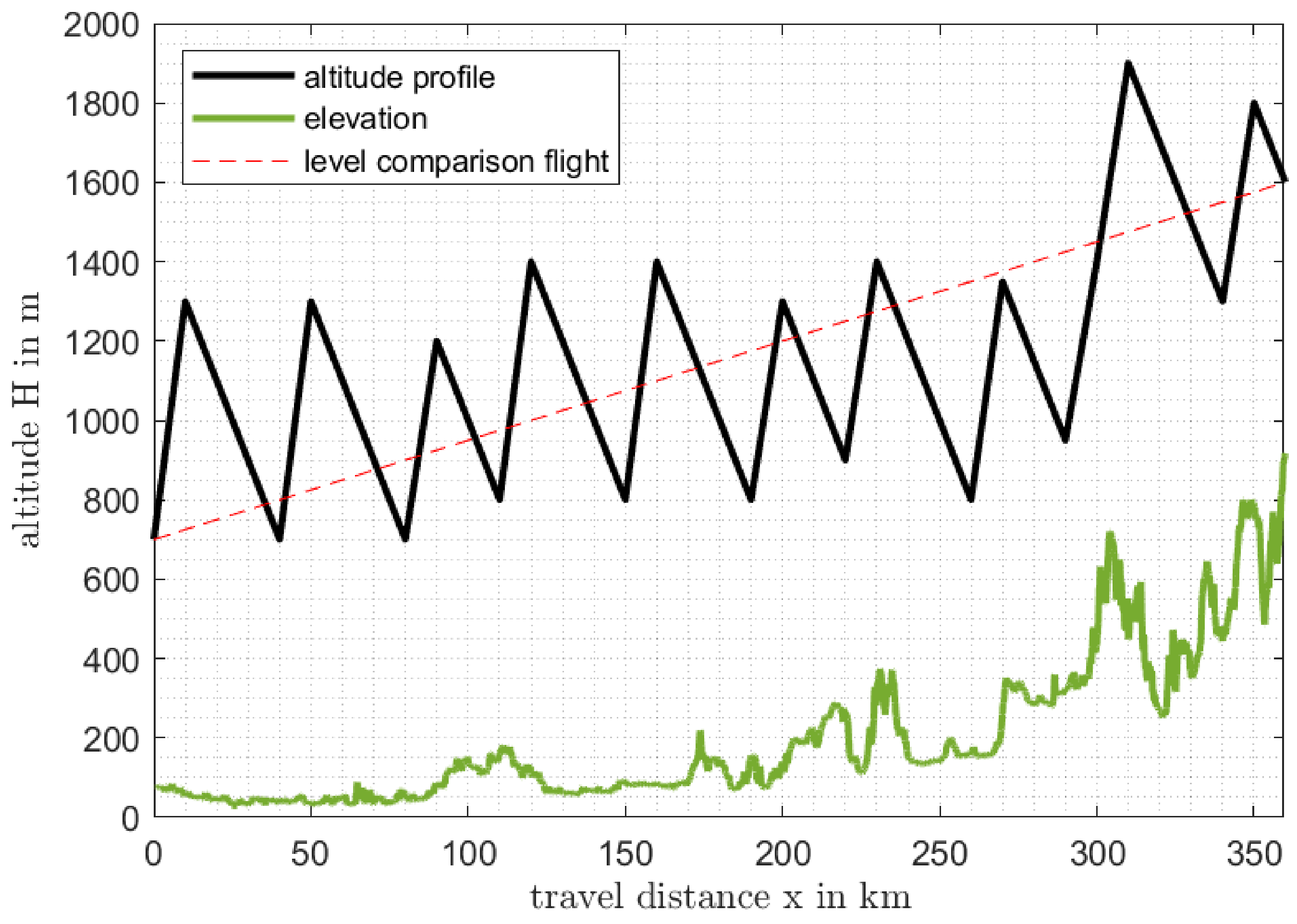
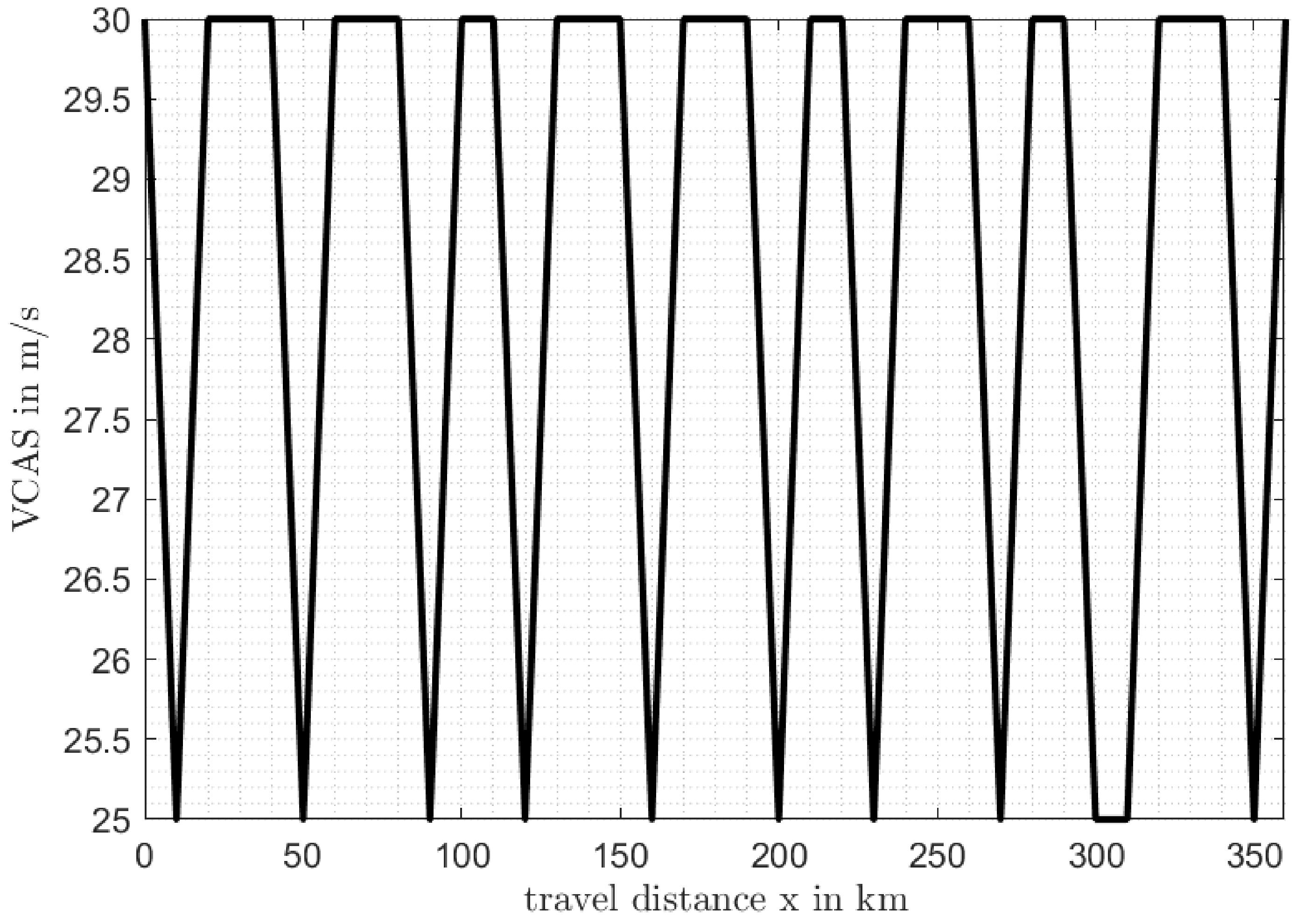
Run dynamic programming: Wind 20m/s East to West

Consumed Energy in optimization: 21.5441 kWh (level: approx 23.1303 kWh)





Run dynamic programming: Wind 20m/s to East:

Consumed Energy in optimization: 56.2873 kWh (level: approx 93.1898 kWh)